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THE THEORY OF THE STRENGTH OF METALS CALCULATED FROM THEIR STRUCTURE

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Introduction

The problem of strength is important in engineering. Up to this time, however, this problem has not been satisfactorily solved in spite of many researchers' having worked on it. At present, various theories of stability exist that establish the critical state of a material for any complex inter-reaction of forces. The fracture process or the phenomenon of plasticity in materials is mainly a function of their structure and of the entire data giving rise to a given detail; they also depend upon the strained state, the form of the element of construction, the nature of the application of stress, the method of conducting tests, temperature, etc. During a state of resistance in these materials, an analytical condition of plasticity is usually set up for an isotropic homogenous and continuous elastic solid, with the introduction of mechanical characteristics of a real material for particular cases of the strained state.

The appearance of new experimental material necessitates introducing new constants into these conditions. Often the theory of plasticity is developed on the basis of clearly formal construction, reflecting only indirectly the physical substance of the phenomenon.

Many of the factors determining stability or the emergence of critical deformations are not amenable to calculation in the accepted plan of an isotropic homogenous body. One of the basic inadequacies of present-day "theories of strength"

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or conditions of plasticity is the failure to take directly into account the structures and states of the materials.

On the other hand, the physics of crystal materials and especially the physics of metal monocrystals have progressed far enough to afford valuable qualitative explanations of the resistance of materials./1/

The problem of setting up conditions governing viscosity in polycrystals for a given strained state must be solved by synthesizing the conditions of viscosity for individual monocrystals. Thus calculations of the crystal structure enable one to obtain an analytical relation between stress and deformation in the plastic region, to establish the original function of the plastic process--the rate of growth of the plastic region along the cross-section relative to the mechanism of deformation--and, further, to find the magnitude of initial stress of the microstructure according to a definite plan./2/

Sachs /3/ first solved the particular problem of determining the yield points for cases of axial tension and distortion, using the method of integration with respect to stereographic projections. In contrast to Sachs' method, we are solving the general problem of determining the yield point in any complex volumetric strained state not only for face-centered, but also for volume-centered cubic and hexagonal lattices, by applying the developed analytical method of simple summation. The bases of the method of averaging (neutralization) are as follows:

1) The principle that the critical shearing stress is constant; the examination of the process of small plastic deformations (the formation of a small region of viscosity).

2) The consideration that the deformation of a polycrystal takes place in the form of a deformation of individual grains. We disregard the interaction on the boundaries of the grains.

3) In calculating the yield points in a complex strained state, the principle of super-position of forces is used.

1. Tangential Stress According to Crystallographic Direction

We assume that the directions and magnitudes of the main stresses $\sigma_z, \sigma_y, \sigma_x$ are known. The coordinate axes z, y, x are parallel to the main stress (sketch). The normal ν to an arbitrarily-placed area makes, with the axes of the coordinates, the angles $(\nu_z), (\nu_y), (\nu_x)$. *Insert sketch here*

We shall find the tangential stress across the area, the normal ν which is ν , in the direction of slip η

Insert formula (1) ~~Eq. 8.55~~ 8.55

We proceed to spherical coordinates, which determine the position of the normal ν , by anti-latitude θ and longitude ϕ , and the position of the direction of shear η by anti-latitude θ' and longitude ϕ' . The relation between the spherical and cartesian coordinates is:

Insert formula (2) 8.56

Inserting these expressions in (1), we obtain a formula

for tangential stress expressed in spherical coordinates

Insert formula

(3) *P. 1*

In this expression we shall separate from the action of the influence of the orientation of the elements of slip on the tangential stress; we shall designate them by

In place of (3) we obtain

(4) ~~*Insert formula*~~

where

Insert formula

It is not difficult to establish the dependence among r_z , r_y , and r_x due to the orthogonality of the directions of the normal ν and slide η

(5)

whence

Insert formula

This relation makes it possible to reduce the problem of determining tangential stress ~~to~~ a volumetric strained state (*Insert formula*) to the problem of calculating it in a plane reduced strained state. We divide the given strained state into the following:

- 1) Thoroughly uniform tension σ ,
- 2) Tension along the two axes and with values of stress σ_1 and σ_2 , whence

(5a)

$$\begin{aligned}\sigma_2' &= \text{Insert formula} \\ \sigma_1' &= \text{Insert formula}\end{aligned}$$

-4-

CONFIDENTIAL

We insert these expressions in (4)

$$\tau_{ij} = \frac{1}{2} \sigma_{ij} \quad (5b)$$

The volumetric strained state gives the same tangential stresses with respect to structural directions as the plane strained state and , for which the differences of the fixed main stresses function correspondingly with respect to the axes x and y.

If the third theory of strength assumes the magnitude of the greatest difference of main stresses to be of primary importance, then, in our investigation of multiple tangential stresses of crystalline grains of the aggregate in the expression , both the largest and the smallest differences of main stresses are included.

In order to obtain the yield points for polycrystals with a random distribution of all crystalline grains, it is necessary to find the combination of all possible values of τ_{ij} for the most diverse orientation of crystals. Generally speaking, it is necessary for an asymmetrical system to have a three-dimensional statistical collection of orientation of crystalline grains, obtained by rotation around the axis z, y, and x. We shall designate these deflection angles of crystalline grains relative to the coordinate axes .

For a cubic lattice, having in mind the high asymmetry of the element of construction, it is completely sufficient to limit ourselves to a two-dimensional collection of orientation (angles α, β, γ); for a hexagonal system, having an element of construction in the form of a six-faced prism,

P.860

it is necessary to base ourselves on the three-dimensional collection of orientation (angles of rotation ...).

We shall investigate in detail the general order of composition of the number of influences of orientations .

2. Determining the Yield Point by the Averaging (Neutralization) Method

Having established the crystallite in the form of its ideal construction on the horizontal plane xy, we rotate it around the axis z through the angles ... etc. For each of these positions of the crystallite, determined by the value of angle ..., we find a series of positions, rotating the crystallite around the axis y. For each of these positions we have in the case of a cubic volume centered lattice, 12 values of tangential stress (six areas of slip with two directions of slip). Let us say that the problem of defining the yield point during tension along the axis z is investigated. For each position of the crystallite we find a maximum tangential stress from all the directions of shear and from all the areas of slip for a given orientation of the crystallite, determined by the angles ... Φ .

We further draw up a table of values τ_{max}^i , proportional to the particular yield points of the aggregate σ_i . We find the general yield points as the average of the arithmetical mean

$$\bar{\sigma} = \frac{1}{n} \sum_{i=1}^n \sigma_i = \frac{1}{n} \sum_{i=1}^n (\tau_{max}^i) \quad (6) \quad \text{p. 860}$$

where τ is the critical shearing stress.

The yield point of the polycrystal with respect to a given small sectional area can be computed as the average of the arithmetical mean out of n particular values of the yield points for individual monocrystals.

In the case of a complex strained state

Insert from (7)

(7) P 160

The conditions of viscosity of a polycrystal will be

Insert from (8)

(8) P 161

That is, the fictitious averaged or "neutralized" tangential stress should be smaller than the critical shearing stress. It does not agree with the "octahedral" tangential stress according to Rosh-Eyklinger; generally, however, it is close to it.

Thus with the help of (8) we found that the yield points of the aggregate for various relations between σ and τ . We shall reduce these results for metals of the iron group and compare with the values obtained according to the conditions of viscosity of Guber-Mises (Table I).

Table II shows the basic results—yield points for three groups of metals (copper, alpha-iron, and zinc), obtained by us by the described calculation, and yield points according to Guber-Mises.

Thus, we have the consequent decrease in influence of the average main stress.

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$$\sigma_s = \frac{1}{n} \sum_{i=1}^n \sigma_i$$

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Thus with the help of (8) we found that the yield points of the aggregate for various relations between σ_s and σ_c . We shall reduce these results for metals of the iron group and compare with the values obtained according to the conditions of viscosity of Guber-Mises (Table I).

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CONCLUSION

As can be seen from the above-stated, the method applied is a numerical method of finding the yield point of a polycrystal metal in any complex strained state for three groups of lattices of alpha-iron, copper, and zinc; also, the influence of structure (Table II) is revealed.

It is established, as a result, that the theory of Guber-Mises gives values of yield points close only to values obtained by us for groups of quasi-isotropic large-grained alpha-iron (upon eliminating the influence of grain boundaries and for regions of small plastic deformations). For fine-grained steel and for cold-hardened material, possessing other plastic properties, the mechanical application of Guber-Mises theory appears to be inadequate.

CITED LITERATURE

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-8- **CONFIDENTIAL**

TABLE I

Comparison with the Theories of Guber-Mises

<i>2.7</i>	
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According to the
Theory of
Crystal StructureAccording to
Guber-Mises*Insert figures* *P. 61*TABLE IIComparison of the Yield Point for Three Groups of Metals
(Copper, Alpha-Iron, and Zinc)

Group	Yield Points		
	Copper Group	Iron Group	Zinc Group
<i>2.7</i>			
			According to Guber-Mises

*Insert figures**P. 61*

- 9 -

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